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Information fusion based approach for studying influence on Twitter using belief theory

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Abstract

Influence in *Twitter* has become recently a hot research topic since this micro-blogging service is widely used to share and disseminate information. Some users are more able than others to influence and persuade peers. Thus, studying most influential users leads to reach a large-scale information diffusion area, something very useful in marketing or political campaigns. In this study, we propose a new approach for multi-level influence assessment on multi-relationnal networks such as *Twitter*. We define a social graph to model the interactions between users as a labeled multi-relational graph where users are represented by nodes, and links model the different interactions between them, namely, *retweets*, *mentions* and *replies*. We explore how interactions between nodes in this graph could reveal about the influence degree and propose a generic computational model to assess influence degree. This is based on the conjunctive combination rule from the belief functions theory in order to combine the different types of interactions. We experiment the proposed method on a large amount of data gathered from *Twitter* during the European Elections 2014 and deduce top influential candidates. The results show that our model is flexible enough to allow multiple interactions combination according to social scientists needs or requirements and that the numerical results of the belief theory are accurate.

Keywords: Social Influence; Information fusion; Multi-level fusion; Belief theory; Twitter network

Introduction

Nowadays, online social networks such as *Twitter* gather people together and empower their relationships with new forms of cooperation and communication. As a result of its massive popularity, *Twitter* is exploited as a platform for very different purposes such as marketing or political campaigns. One of the most distinctive characteristics of *Twitter* is the information diffusion through social links. In fact, links between users impact the information flow and thus indicate the user's influence on others. Some users, called influentials, are more able than others to diffuse information to a huge number of users. Therefore, determining influential users in a network is a secret key of success for achieving a large scale information diffusion at low cost.

The Merriam-Webster dictionary has defined influence as " The power or capacity of causing an effect in indirect or intangible ways." Despite the large number of influence theories in sociology, there is no tangible way to measure such a power. Focusing on an individual's potential to engage others in a certain act, Alex et al. [1] have defined influence on *Twitter* as the potential of a user's action to initiate a further action by another user. The term "action" means the different possible interactions between users. Hence, measuring influence on *Twitter* is not that simple as the application provides several forms of interactions. A user can *follow* another one, which allows him to see *tweets* and information about the user he follows. He is also able to *retweet* a *tweet*, this exposes the *tweet* to his followers who can also *retweet* it. A user can *mention* another one by using the "@" prefix if he wants to address or to show him the *tweet*. And finally, a user can *reply* to another's *tweet* and thus creates a conversation with him. These different interaction patterns are what made *Twitter* a multi-relational network [2, 3] on which possible links can be *retweet*, *mention*, *reply* or *follow*. While measuring influence, the choice of these actions depends on understanding the subject and domain area [4].

Influence assessment poses three main challenges. The first is the diversity of actions on which we can rely to compute influence. Morevover, joint use of actions in a *tweet* can have different meanings. For example using several mentions of media at the end of a tweet allows to expose the *tweet* in the largest number of twittos. It is important to combine actions in order to establish a general influence measure that takes into account the different types of interactions among users. The second is the consideration of indirect influence. In some cases, the influence is not direct, it extends to a user through intermediates users. For example, a user may *retweet* another's *tweet* indirectly through an intermediate user. It is necessary to measure the influence regarding the direct and indirect interactions in the network. The third challenge is related to uncertainty when the combination of actions is performed. In the case of multi-relational networks, due to the multiple semantics of interactions, it is difficult to assign importance weights to the different actions before merging their related quantitative data.

In this paper, we extend the model proposed in [5] and propose an algorithm for multi-level information fusion with interaction patterns. Our contributions are manifold. In order to evaluate influence, we define a social graph allowing us to capture the relationships between users as a labeled multiplex graph where users are represented by nodes, and links model the different interactions between them. We also combine the different actions obtained from the network to interaction patterns. The measure can be established between a couple of users by taking into account different actions or patterns between them or it may also assess a user's global influence in the network considering all the actions or patterns where his peers are involved in. We also consider uncertainty in the measurement process. We define a theoritical framework, to compute influence, based on the conjunctive combination rule for belief functions theory and Smets rule [6] to fusion and combine information about different actions. The proposed approach is flexible thus indirect influence in multi-relationnal graph can also be taken into consideration. In this case, the influence measure consider influence exercised on indirected nodes (e.g. a user may *retweet* another's *tweet* indirectly through an intermediate user). This spans the influence on multiple levels based on a multi-level diffusion network. An evaluation through experiments is proposed. It is based on real data gathered from Twitter in the TEE 2014 project during the European Election campaign in 2014.

The rest of the paper is organized as follows. Section 2 presents literature review. Section 3 describes our proposed approach. Section 4 presents the experimental results. And finally section 5 concludes the paper.

Literature review

In this section, we review studies of influence assessment in *Twitter* and remind the basic concepts of belief functions theory on which our approach is based.

Influence in *Twitter*

Regardless of the social network studied, assessing user's influence in the network is a major concern due to the social networks massive popularity and their important role in information diffusion [7]. Nowadays, detecting and ranking influential users are a key secret of information propagation through social links [8]. Researchers have been interested in assessing influence in social networks and many approches were provided to rank users according to their influence. Some approaches are based on **network topology** and centrality measures [9]. Others approaches try to establish a ranking of nodes by using **diffusion-based** or random-walk based algorithms like PageRank [10] or HITS [11] algorithms. A novel family, using belief theory, extends network topology approaches to take into account **information fusion**. In the next subsections, we present major works on *twitter* data-network for each approach type.

While measuring users influence in *Twitter*, many criteria can be considered. Leavitt *et al.* in [1] use three features to measure influence, which are: *replies, retweets* and *mentions* in addition to number of *followers*. They give statistics related to these measures and do not offer a global influence score based on all the proposed actions. Cha *et al.* [12] define three influence measures in *Twitter*, the indegree influence, which is the number of followers, indicating the size of a user's audience or popularity; the *mention* influence corresponds to the number of a user's *mentions*, indicating his ability to engage others in *mention*; and the *retweet* influence, which is the number of *retweets*, indicating the ability of a user to write content to be forwarded to others. The authors compute the value of each action for 6 million users and compare them. In order to do this, they sort users according to each different action, after that, they quantify how a user's rank varies across different actions. Spearman's rank correlation is used as a measure of the association strength between two rank sets. They found that *followers* number represents a user's popularity, but is not related to other important actions such as *retweets* and *mentions*. Their result suggests that *followers* number alone reveals very little about a user's influence, this is also know as *the million follower fallacy*. This research does not provide a global influence measure and only influence measures according to each action separately. Chen *et al.* [13] propose a local ranking method named ClusterRank, which takes into account the number of neighbors and the clustering coefficient. Bakshy *et al.* [14] followed a different approach to estimate influential users: they use shortned URL diffusion cascades, users producing the largest cascades are the most influential. Brown *et al.* in [15] believe that the location of a node in the network may play a more important role than its indegree. For example, a node located in the center of the network, having few highly influential neighbors, may be more influential than a node having a larger number of less influentials neighbors. Considering this fact, *k-shell* decomposition algorithm can be useful [16]. Basically, the principle of the *k-shell decomposition* is to assign a core index *ks* to each node such that nodes with the lowest values are located at the periphery of the network

while nodes with the highest values are located in the center of the network. The innermost nodes thus form the core of the network. They observe that the results of the *k-shell* decomposition on *Twitter* network are highly skewed. Therefore they propose a modified algorithm that uses a logarithmic mapping, in order to produce fewer and more meaningful *k-shell* values.
The disadvantage of the network topology based algorithms is to consider the attribute information of the user, and not to consider the interaction among users through a sequence of operators. In *Twitter*, the user's influence is impacted by the information diffusion between the users. Correlation between users actions were considered in [17] to identify and measure social influence as a source of correlation between the individuals actions with social ties. Authors study the phenomenon that a user's actions can induce his/her friends to behave in similar way. To do this, they use logistic regression to quantify social correlation. This is measured as a function of only one variable: the number of active friends the user has. After this, the *shuffle test* is used to decide if influence is a likely source of correlation. The techniques used provide only a qualitative indication of the influence existence and not a quantitative measure. Qasem *et al.* [18] presented a new approach of influential users detection. The proposed approach detects the users who increase the size of social network by attracting new users into the network.

Other researches propose to rank nodes by using **diffusion-based** or random-walk based algorithms, with a common assumption that a node is expected to be influencial if it points to many highly influential neighbors. In this context, user's influence were ranked based on the classical random walk algorithm such as PageRank. The main idea behind PageRank is that "more important pages (web sites) are likely to receive more links from other pages". Many variants of the PageRank algorithm were proposed in order to improve it and adapt it to *Twitter*. A notable one was TunkRank [19], it uses a constant to represent the retweet probability, combined with the people whom the user concerned and the fans who concerned this user. The user's influence was the expected number of the people influenced by the released information in TunkRank. Gosh *et al.* [20] propose Collusionrank, a PageRank-like approach, to overcome link farming in *Twitter*. They negatively bias the initial scores towards nodes identified as spammers. Then, since a user should be penalized for following spammers and not for being followed by spammers, the Collusionrank score of a node is computed based on the score of its followings (instead of its followers). Thus, users who follow a larger number of spammers, or who follow those who in turn follow spammers, get a negative score of higher magnitude and are pushed down in the ranking. On the basis of PageRank, LeaderRank [21] introduces a ground node g, which has two directed links to every node in the original network, so that the network becomes strongly connected. LeaderRank converges faster since the network is strongly connected. The results showed that LeaderRank outperformed PageRank in terms of ranking effectivenes, as well as robustness against manipulations and noisy data. Li *et al.* [22] improve LeaderRank by introducing a weighted mechanism: nodes with different in-degrees get different ranks from the ground node. Wang *et al.* [23] propose PhysarumSpreader by combining LeaderRank with a positive feedback mechanism inspired from an amoeboid

organism called Physarum Polycephalum. In [24], authors define a measure based on topic similarity and structure in the links between users. Influence is considered as the fact of *following* other users regarding topic interests. In this context, the authors propose TwitterRank, an extension of the PageRank algorithm, in order to measure the topic-sensitive influence of users. Although the idea is promising, the experimental results show that there are some *follow* links between users not because of the topic similarity between them, also the method ignored other important criteria such as *mentions* and *replies*.
In the same context of the diffusion based algorithms, some researchers proposed variants of the Hyperlink-Induced Topic Search (HITS) a link analysis algorithm that rates Web pages, developed by Jon Kleinberg. HITS assigns two scores for each page: its authority, which estimates the value of the content of the page, and its hub value, which estimates the value of its links to other pages. Romero *et al.* [25] propose the IP-Algorithm in order to measure influence. In this paper, influence is considered as the degree of content propagation in the network (*retweets*). In addition, authors believe that a user's influence depends not only on the size of the influenced audience, but also on their passivity. The passivity of a user is his passive information consuming without forwarding the content to the network. The algorithm showed better accuracy than other influence measures such as PageRank, the number of *followers* and number of *mentions*. Although passivity seems a good influence indicator, this work ignored other important influence marker such as *reply*.
The diffusion based algorithms such as variants of the PageRank and HITS were designed considering the information propagation in the network. Their shortcoming is the lack of actions combination.

In recent works, **information fusion** is considered in order to address limitations of existing methods. In [26], authors propose a combination of two models for ranking users' influence: The PageRank algorithm [10] and HMM (Hidden Markov Model). They build a HMM to observe the influence evolution over time and use three observables: *retweet*, *mention* and *reply*. The model is evaluated using a survey as ground-truth for influence ranking. The proposed model differs from the others by combining the important influence markers. However, as the purpose is to rank users' influence, a user's given influence does not reveal information about its influence degree (high or low influence), the model's output is only useful in users ranking.

Existing research proposes methods to measure influence, however, none of them presents an approach for *Twitter* influence regarding multi-criteria combination, also, uncertainty has not been considered yet in such combinations. It is important to assess influence taking into account the degrees of uncertainty about the weights assigned to different criteria according to their importance. In this purpose, we propose the use of belief functions theory. In recent researches, belief functions theory were exploited to asses user's influence in weighted networks [27, 28] and complex networks [29] with the common goal of modifying the existing evidential centrality. To the best of our knowledge, this is the first time belief functions theory is exploited to asses influence on *Twitter* network with interaction patterns instead of centrality measures.

Belief functions theory

Every day, a huge volume of incomplete and imperfect information is produced by social networks applications. Thus, reasoning with uncertainty has become a major interest in the analysis of social networks data.

The belief functions theory is considered as a general framework for reasoning with uncertainty, and has well been connected to other frameworks such as probability, possibility and imprecise probability theories [30]. The theory of belief functions, also known as evidence theory or Dempster-Shafer theory, was first introduced by A. Dempster in the context of statistical inference, and was later developed by G. Shafer as a general framework for modeling epistemic uncertainty [31].

In the following paragraph, we are going to remind the basic concepts of belief functions theory. Let Ω be a finite set, denote by 2^{Ω} the set of all subsets of Ω. In the context of Dempster-Shafer theory, Ω, often called a frame of discernment, represents the set of possible hypotheses. A mass m is a function $m : 2^{\Omega} \longrightarrow [0,1]$ such that:

$$\sum_{X \in 2^{\Omega}} m(X) = 1 \text{ and } m(\emptyset) = 0 \tag{1}$$

The mass $m(X)$ expresses the part of belief that supports the subset X of Ω. According the theory of closed-world, Ω is exhaustive, hypotheses are mutually exclusive and $m(\emptyset) = 0$.

Belief functions theory allows, not only the representation of the partial knowledge, but also the information fusion under uncertainty [32]. This is done by the conjunctive combination rule [6], it assumes that all sources are reliable and consistent. Considering two mass functions m_1 and m_2, the conjunctive combination rule is defined as:

$$(m_1 \bigcirc\!\!\!\!\cap\, m_2)(C) = \sum_{A \cap B = C} m_1(A) m_2(B), \quad A, B, C \in 2^{\Omega} \tag{2}$$

In order to make a decision, we try to select the most likely hypothesis which may be difficult to realize directly with the basics of the belief functions theory where mass functions are given not only to singletons but also to subsets of hypothesis. There exist several solutions to ensure decision making within belief functions theory. The most known is the pignistic probability [33]. In contrast to mass functions that are defined on 2^{Ω}, pignistic probability is a probability measure defined on Ω. Pignistic probability was proposed in the Transferable Belief Model (TBM) [34]. It is based on two levels: The "credal level" where beliefs are entertained and represented by belief functions and the "pignistic level" where beliefs are used to make decisions and represented as probability functions called pignistic probabilities denoted *bet*:

$$\text{bet}(x) = \sum_{x \in X \subseteq \Omega} \frac{m(X)}{|X|} \tag{3}$$

Belief functions theory has been widely used in many fields such as natural risks [35]. To the best of our knowledge, this is the first time belief functions are exploited in influence assessment on *Twitter*.

Proposed approach

In order to assess users' influence, we propose a belief approach based on information fusion about the different possible influence relationships or interaction patterns. Figure 1 gives an overview of the framework for the proposed approach. In the first two steps, information from Twitter network is modeled in a graph by selecting relevant relationships or patterns for the influence model. In the two following steps, at a credal level, we associate belief masses for each relationship and we combine them to obtain the influence belief mass. At the pignistic level, we compute the pignistic probability in order to make a decision about the user's influence degree. In the following section, we detail each step of the assessment process.

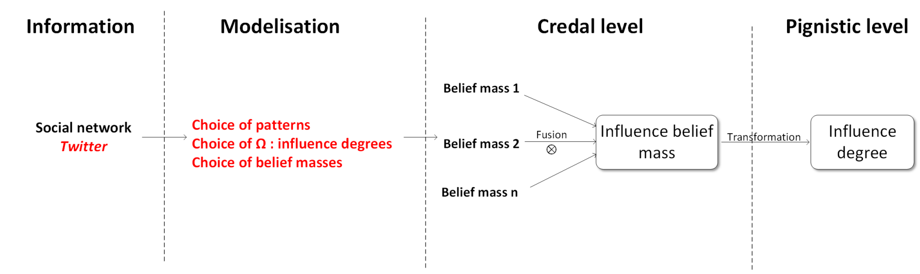

Figure 1 Framework of the proposed approach

Belief diffusion graph

Social networks have been widely modeled as a graph [36]. A graph is represented as $G = (V, E)$ comprising a set V of vertices or nodes together with a set E of edges or links. In *Twitter* network, the graph is heterogeneous as we have many relationships between nodes and different types of nodes. For example, there may exist a link *follower* between two users, a link *retweet* between one *tweet* and a user. In order to model this heterogeneity, we use a multi-relationnel graph [3]. As we want to evaluate a user's influence on other users, we restrict the graph to homogenous nodes (users), thus, we have a multiplex graph (sometimes called multi-layered graph) [37]. In a multiplex graph the set of links E is divided into pairwise disjoint classes $E = \bigcup_{r \in R} E_r$, where R is the set of possible relationships. For example, in *Twitter* we can consider the following relationships R = {Retweet, Mention, Reply, Follow} inferred by the use of twitter' actions like RT, @.

We define an interaction pattern p as a sequence of relationships, for example a *retweet* of a *reply* or *retweet* of a *tweet* with a *mention*. Let P be the set of all the interaction patterns that have been identified for modelling influence in a specific domain or for a specific study. This set must be given by social scientists for example.

We denote by $R = R \bigcup P$ the set of relationships including multiple level relationships (i.e. interaction pattern). In this context, we introduce the belief diffusion graph (Figure 2) as a labeled multiplex graph where nodes are represented by users, and links model the different relationships between them. The links are labeled with influence degrees (e.g., Weak, Average, Strong) and belief masses that depend on the type of the relationships. Uncertainty is injected intentionally for evaluating influence. Nodes are labeled with uncertainty about their estimated influence degree resulting from the fusion of the belief masses of incident links. Some recent researches have introduced uncertain graphs whose edges are labeled with a probability of existence [38,39]. In our case uncertainty is not about the presence or absence of links but is about the semantics weight of links according to the domain. For example in political studies, a mention or a reply can be less valuable than a retweet, also a reply followed by a retweet is a very important pattern.

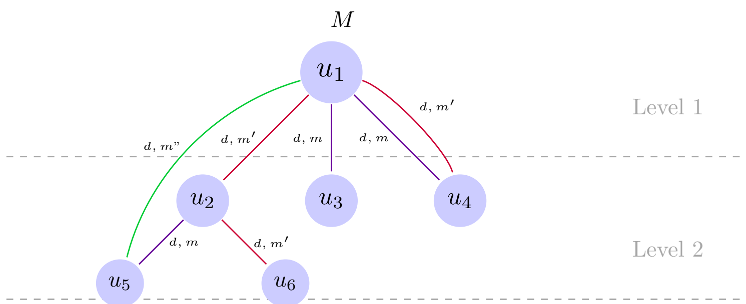

Figure 2 Belief diffusion graph. Nodes represent users, and links are different relationships between them. Relationships are represented by color red and purple and pattern are represented in green. Different masses m and m' in this example are associated with each relationship. The mass $m"$ is computed for the interaction pattern between node $u1$ and node $u5$ through the intermediate node $u2$. d is the influence degree associated with each link. Using a combination procedure, discussed below, we obtain a global influence (denoted by M) of node u_1.

Masses fusion on belief diffusion graph

Based on the Dempster-Shafer theory discussed above in the literature review section, we explain how to make the fusion of different mass functions defined on the belief diffusion graph.

Let Ω be an ordered set of possible influence degrees:

$$\begin{aligned}\Omega = \{&\text{Very Weak}, \text{Weak}, \text{Average Enough}, \text{Average}, \text{Strong Enough},\\ &\text{Strong}, \text{Very Strong}, \text{Extremely Strong}\}\end{aligned} \tag{4}$$

In general Dempster-Shafer theory we should use 2^{Ω} as a domain of mass functions. But in our approach we use only a certain subset Λ of 2^{Ω}, precisely:

$$\begin{aligned}\Lambda = \{&\text{Very Weak}, \text{Weak}, \text{Average Enough}, \text{Average}, \text{Strong Enough},\\ &\text{Strong}, \text{Very Strong}, \text{Extremely Strong}, \Omega\}\end{aligned} \tag{5}$$

A mass function is associated for each relationship, and so, mass functions are defined as follows:

$$m_r : \Lambda \to [0, 1] \tag{6}$$

Table 1 Definition of the operation @

@	**V.Weak**	**Weak**	**Average.E**	**Average**	**Strong.E**	**Strong**	**V.Strong**	**E.Strong**	**Ω**
V.Weak	Weak	Average.E	Average	Strong.E	Strong	V.Strong	V.Strong	E.Strong	V.Weak
Weak	Average.E	Average.E	Average	Strong.E	Strong	V.Strong	V.Strong	E.Strong	Weak
Average.E	Average	Average	Strong.E	Strong	V.Strong	V.Strong	V.Strong	E.Strong	Average.E
Average	Strong.E	Strong.E	Strong	Strong	V.Strong	V.Strong	V.Strong	E.Strong	Average
Strong.E	Strong	Strong	V.Strong	V.Strong	V.Strong	V.Strong	V.Strong	E.Strong	Strong.E
Strong	V.Strong	V.Strong	V.Strong	V.Strong	V.Strong	E.Strong	E.Strong	E.Strong	Strong
V.Strong	V.Strong	V.Strong	V.Strong	V.Strong	V.Strong	E.Strong	E.Strong	E.Strong	V.Strong
E.Strong	E.Strong	E.Strong	E.Strong	E.Strong	E.Strong	E.Strong	E.Strong	E.Strong	E.Strong
Ω	V.Weak	Weak	Average.E	Average	Strong.E	Strong	V.Strong	E.Strong	Ω

In order to estimate the influence degree of a specific node u, we take into account the local structure of the belief diffusion graph around the node u and combine the belief mass functions of incident links using a modified version of conjunctive combination rule (2).

$$(m \otimes m')(z) = \sum_{y\,@\,x=z} m(x)m'(y), \quad x, y, z \in \Lambda \tag{7}$$

where @ is a symmetric operation $@ : \Lambda \times \Lambda \to \Lambda$. Table 1 represents an example of an @ operation.

Proposition 1 *A combination of any two mass functions is another mass function.*

Proof. Denote $(m \otimes m')$ by m''. It is easy to see that for all x we have $m''(x) \geqslant 0$, because we compute m'' using only multiplication and addition of non-negative numbers. Next, we show that $\sum_{z\in\Lambda} m''(z) = 1$. Let $\Lambda_z^2 = \{(x, y) \in \Lambda : x@y = z\}$ and proceed as follows:

$$\begin{aligned}\sum_z m''(z) &= \sum_z \sum_{x\,@\,y=z} m(x)m'(y)\\ &= \sum_z \sum_{(x,y)\in\Lambda_z^2} m(x)m'(y)\,.\end{aligned}$$

Note that $\Lambda_z^2 \neq \Lambda_{z'}^2 \iff z \neq z'$, and $\bigcup_{z\in\Lambda} \Lambda_z^2 = \Lambda^2$. So, we can ommit $\sum_z$ and rewrite as follows:

$$\begin{aligned}&= \sum_{(x,y)\in\Lambda^2} m(x)m'(y)\\ &= \sum_x \sum_y m(x)m'(y)\\ &= \sum_x m(x) \sum_y m'(y)\end{aligned}$$

m and m' are mass function: $\sum_x m(x) = \sum_y m'(y) = 1$, so $\sum_x m(x) \sum_y m'(y) = 1$ $\square$.

Proposition 2 *In general* $\otimes$ *is non-associative:* $(m \otimes m') \otimes m'' \neq m \otimes (m' \otimes m'')$

Proof. Consider $\Omega = \{A, B, C\}$, and the following @:

@	***A***	***B***	***C***	Ω
A	B	B	C	A
B	B	C	C	B
C	C	C	C	C
Ω	A	B	C	Ω

$$m = m' = \begin{array}{|c|c|c|c} A & B & C & \Omega \\ \hline 1 & 0 & 0 & 0 \end{array}$$

$$m'' = \begin{array}{|c|c|c|c} A & B & C & \Omega \\ \hline 0 & 1 & 0 & 0 \end{array}$$

It's easy to see that:

$$(m \otimes m') \otimes m'' = \begin{array}{|c|c|c|c} A & B & C & \Omega \\ \hline 0 & 0 & 1 & 0 \end{array}$$

$$m \otimes (m' \otimes m'') = \begin{array}{|c|c|c|c} A & B & C & \Omega \\ \hline 0 & 1 & 0 & 0 \end{array}$$

Thus, in general:

$$(m \otimes m') \otimes m'' \neq m \otimes (m' \otimes m'')$$

$\square$.

Now, we consider multiple interactions existing between node u and its neighbors. We associate a mass function m_r to any relationship $r \in R$. We denote by I_r the set of all links with relationship type r. Finally, we have the following set of mass functions $\{m_{r,i} : r \in R, i \in I_r\}$. Based on Prop. 1 we can combine these mass functions in order to obtain a global belief mass corresponding to the influence degree of node u. But the order of combinations may affect our results (Prop. 2). To be consistent in our measurements we have to choose the order. In order to simplify the expressions we will write $\bigotimes_{i\in\{1,2,3\}}$ instead of $m_1 \otimes m_2 \otimes m_3$. Thus, we consider the following order of combinations:

1 For a given relationship r we subsequently combine the masses of r in order to get r-preresult with $\hat{m}_r$ defined as follows: $\hat{m}_r = \bigotimes\limits_{i\in I_r} m_{r,i}$

2 Then we combine all r-preresults using: $\bigotimes\limits_{r\in R} \hat{m}_r$

Depending on the operation @ such procedure may finally converge to certain stationary mass.

Pignistic probability transformation

Once we have the global belief mass on a certain node, we use a modified version of the pignistic probability defined on equation 3 in order to make the decision about the influence degree of a user. In our case the belief masses is defined on Λ and the pignistic probability is calculated by distributing uniformly the mass of Ω to all other elements of Λ:

$$\text{bet}(x) = m(x) + \frac{m(\Omega)}{|\Omega|}, \quad x \in \Omega \tag{8}$$

The process of the proposed approach is decribed in algorithm 1, the algorithm requires as input, the set of users U, the set of relationships R, the multiplex graph G on U and R, the masses and influence degree initialization for the different relationships, and the operation $@$. For each user, the algorithm starts by counting the number of occurences for each relationship or pattern. Then, for each relationship r, using the equation 7, it computes the belief masses combination. After that, equation 7 is used again to combine the belief masses for all relationships. And finally, using equation 8, the belief masses distributions are transformed to pignistic probability. The algorithm returns the final influence degree which is the degree having the maximal pignistic probability. The source code is available on github[1].

Algorithm 1: Masses fusion on belief diffusion graph

Input : Set of users U.
Set of relationships R.
G the multiplex graph on U and R.
Masses initialization.
Operation $@$

Output: Influence degrees Inf_U

for $u \in U$ **do**
    for $r \in R$ **do**
        count in G the number of relationships or patterns ;
        compute $Inf_{u,r}$ influence belief masses combination using (7);
    end
    compute Inf_u using (7) ;
    compute pignistic probability using (8);
    $Inf_U[u]$= Influence degree having maximal pignistic probability ;
end
return Inf_U ;

Illustrations

In order to illustrate our method, we remind the proposed approach process on an example, we consider the following mass functions initialization associated to the relationships:

$$\text{Retweet} \mapsto \begin{cases} m_{\text{retweet}}(\text{Weak}) = 0.4 \\ m_{\text{retweet}}(\Omega) = 0.6 \end{cases} \qquad \text{Mention} \mapsto \begin{cases} m_{\text{mention}}(\text{V.Weak}) = 0.3 \\ m_{\text{mention}}(\Omega) = 0.7 \end{cases}$$

The belief masses $m_{\text{retweet}}(\Omega)$ and $m_{\text{mention}}(\Omega)$ represent the partial ignorance.

Case 1: Two retweets

[1] https://github.com/kerzol/Influence-assessment-in-twitter

After initialisation of belief masses on the different relationships, we follow the proposed approach process to measure the influence resulted from combination of two *retweets* from one user to another. We first use the operation @ giving the correspondances between the influence masses, then we calculate the conjunctive combination. The combined mass function of the two *retweets* are shown in table 2:

Table 2 Combination of two *retweets*

$\otimes$	**Weak** 0.4	**Ω** 0.6
Weak 0.4	Average.E 0.16	Weak 0.24
Ω 0.6	Weak 0.24	Ω 0.36

We obtain then:
$m(\text{Weak}) = 0.24 + 0.24 = 0.48$
$m(\text{Average.E}) = 0.16$
$m(\Omega) = 0.36$

Finally, to make a decision on the influence degree, we calculate the pignistic probability using equation 8 (Table 3). For example, for the degree Weak, we proceed as follows to obtain the pignistic probability:

$$\text{bet}(\text{Weak}) = m(\text{Weak}) + \frac{m(\Omega)}{|\Omega|} = 0.48 + \frac{0.36}{8} = 0.525$$

Table 3 Pignistic probability for two retweets

V.Weak	0.045
Weak	0.525
Average.E	0.205
Average	0.045
Strong.E	0.045
Strong	0.045
V.Strong	0.045
E.Strong	0.045

We conclude that the influence degree is Weak since it has the highest pignistic probability 0.525. This latter was 0.4 before considering the combination.

Case 2: 2 retweets + 2 mentions

In the second case, we consider two additional *mentions* existing between the same users of case 1. In order to measure influence, we use our proposed process to combine masses of the two *mentions* then we combine the obtained massess with the results of the previous case related to two *retweets* combination.

The conjunctive combination on the two *mentions* gives:

Table 4 Combination of two *mentions*

$\otimes$	**V.Weak** 0.3	**Ω** 0.7
V.Weak 0.3	Weak 0.09	V.Weak 0.21
Ω 0.7	V.Weak 0.21	Ω 0.49

We obtain:
$m(\text{V.Weak}) = 0.42$
$m(\text{Weak}) = 0.09$
$m(\Omega) = 0.49$

Now, we combine the obtained masses with the results of the case 1:

Table 5 Case 2: 2 *retweets* + 2 *mentions*

$\otimes$	**Weak** 0.48	**Average.E** 0.16	**Ω** 0.36
V.Weak 0.42	Average.E 0.2016	Average 0.0672	V.Weak 0.1512
Weak 0.09	Average.E 0.0432	Average 0.0144	Weak 0.0324
Ω 0.49	Weak 0.2352	Average.E 0.0784	Ω_{Inf} 0.1764

We obtain:

$m(\text{V.Weak}) = 0.1512$

$m(\text{Weak}) = 0.2676$

$m(\text{Average.E}) = 0.3232$

$m(\text{Average}) = 0.0816$

$m(\Omega) = 0.1764$

We note that, by combining the four relationships, the belief mass on the degree Weak has decreased compared to the first case, this is due to the fact that the mass of the Average.E degree has increased and became equal to 0.3232. We also notice that the degree Average appeared with a mass equal to 0.0816. We can conclude that the more we have interaction patterns and the more we combine them, the highest influence we get.

Now to make the decision on the influence degree, we compute the pignistic probability (table 6). We conclude that the influence degree for 2 retweets and 2 mentions is Average.E with a pignistic probability of 0.34525. This latter was 0.205 before considering the two *mentions*.

Table 6 Pignistic probability for case 2

V.Weak	0.17325
Weak	0.0.28965
Average.E	0.34525
Average	0.10365
Strong.E	0.02205
Strong	0.02205
V.Strong	0.02205
E.Strong	0.02205

Based on these examples, it is easy to compute the influence for a pattern "retweet + mention" used two times. We follow the same procedure and consider that the pattern is a relationship.

Experiments and results

The research work takes place in the project TEE 2014 whose exact title is: "*Twitter* in the European Elections: An international contrastive study of *Tweets* use by candidates in elections to the European Parliament in May 2014". This international project led by the House of Human Sciences (MSH) in Dijon, brings together nearly 45 researchers (political scientists, sociologists, communication researchers and computer scientists), 10 research laboratories spread across 6 European countries (France, Germany, Belgium, Italy, Spain and the UK). The overall objective of this project is to observe and analyze the *Tweets* communication policies during the election period in May 2014 in various countries of Western Europe.

Data description

The *tweets* collection during the election period has build a corpus which is then analyzed. To collect information from *Twitter*, we used our developed tool *SNFreezer*[2] [40]. The purpose of gathering is to retrieve *tweets mentioning* designated users, those containing a *hashtag*, a word or phrase, or *tweets* sent by candidates. Three types of information (generalized under the term "source") are taken as a parameter to query *Twitter* : user accounts; *hashtags* and words or phrases.

These sources were chosen by political scientists, and among them we find the names of the leading candidates, their *Twitter* accounts and their parties. The collection allowed us to have a large number of tweets (37 million) retrieved for 50 consecutive days, and to massively process these data.

Experiments and results

Our experimental goal is to measure candidates influence on the network. Unlike illustrations given in the previous section, we do not consider the case of measuring influence between two users but rather global candidates' influence in the network. Also, the measure takes into account both the direct and indirect influence.

Direct influence

The direct influence (first-level influence) takes into account only the direct links between the users in the diffusion network. Relationships representing the direct influence are *retweets*, *mentions* and *replies*. To measure influence on the first level, we affect masses to the considered relationships, then we take each candidate's number of *retweets*, *mentions* and *replies* and combine their masses.

The choice and the affectation of the masses in the initialisation step is an important issue while dealing with real data. In some domains such as politics, users have very high number of interactions. When we used the masses initialized in the illustration's section on our data, influence converges to the highest influence E.Strong after a short number of iterations ($\simeq 45$ iterations). In this way, we can not compare candidates' influence as we obtain the same influence with similar masses for most of them. To deal with this, we perform a rescaling and use the following masses initialization:

$$Retweet \mapsto \begin{cases} m_{\text{retweet}}(\text{V.Weak}) = 0.55 \cdot 10^{-3} \\ m_{\text{retweet}}(\Omega) = 1 - 0.55 \cdot 10^{-3} \end{cases}$$

$$Mention \mapsto \begin{cases} m_{\text{mention}}(\text{V.Weak}) = 0.45 \cdot 10^{-3} \\ m_{\text{mention}}(\Omega) = 1 - 0.45 \cdot 10^{-3} \end{cases}$$

$$Reply \mapsto \begin{cases} m_{\text{reply}}(\text{V.Weak}) = 0.45 \cdot 10^{-3} \\ m_{\text{reply}}(\Omega) = 1 - 0.45 \cdot 10^{-3} \end{cases}$$

Tables 7 and 8 show the first level combination results for the French candidates "Marine Le Pen", "Florian Philippot" and "Jean-Luc Mélenchon" and the English

[2] https://github.com/SNFreezer

candidates “Katie Hopkins”, “Nigel Farage” and “Patrick O’Flynn”. For example, we conclude that the influence degree for the candidate “Marine Le Pen” who has 14678 *retweets*, 66798 *mentions* and 4003 *replies*, is E.Strong with the belief mass of 0.8173448. Results given do not only provide the influence degree but also give indication of our belief in the given results which is performed by the belief masses on the different degrees.

Table 7 Results for the top 3 influential French candidates

	M. Le Pen	**F. Philippot**	**J.L. Mélenchon**
E.Weak	0	0.000011065	0.000030278
Weak	0	0.00007295998	0.0001832843
Average E	0	0.0007035528	0.001403947
Average	0	0.003033557	0.004954501
Strong.E	0	0.008340205	0.01247841
Strong	0	0.02191526	0.02977818
V.Strong	0.1826552	0.5830090	0.7960571
E.Strong	0.8173448	0.3829144	0.1551143
Ω	0	0	0

Table 8 Results for the top 3 influential English candidates

	Katie Hopkins	**Nigel Farage**	**Patrick O’Flynn**
E.Weak	0	0	0.0022350807
Weak	0	0	0.0067817003
Average E	0	0	0.0292114368
Average	0	0	0.0707907792
Strong.E	0	0	0.1140117014
Strong	0	0	0.1635190125
V.Strong	0.0260529	0.2663876	0.5804666241
E.Strong	0.9739471	0.7336124	0.0325333152
Ω	0	0	0.0004503499

Multi-level belief fusion with interaction patterns

In the previous results, we only considered direct influence for the candidates. The proposed approach is also flexible and can be extended to multi-level belief fusion using patterns. This means that indirect influence can be considered. For example, we can consider the *retweets* of *replies* or the *retweets* of the *mentions*. Those relations are called indirect relathionships as they are performed on indirect nodes in the network (e.g. a user may *retweet* another’s *tweet* indirectly through an intermediate user). The indirect influence is a more complex form of dialog compared to the direct influence. Relationships or patterns are determined depending on the domain.

In order to consider indirect influence in our assessment, we evaluate the indirect influence and then combine the results with those obtained from the direct influence. We affect the following masses to the new patterns *Retweet of Reply* and *Retweet of Mention*:

$$\textit{Retweet of Reply} \mapsto \begin{cases} m_{\text{retweetOFreply}}(\text{V.Weak}) = 0.75 \cdot 10^{-3} \\ m_{\text{retweetOFreply}}(\Omega) = 1 - 0.75 \cdot 10^{-3} \end{cases}$$

$$Retweet\ of\ Mention \mapsto \begin{cases} m_{\text{retweetOFmention}}(\text{V.Weak}) = 0.65 \cdot 10^{-3} \\ m_{\text{retweetOFmention}}(\Omega) = 1 - 0.65 \cdot 10^{-3} \end{cases}$$

The masses are a little more important than those intitiated for the direct influence interactions because we consider that indirect interactions are good indicator of influence, this shows that some users are able to diffuse *tweets* on many levels and they are exercing influence even on users with which they are not connected.

Table 9 represents the results of the multi-level belief fusion for top 3 influencial French candidates. It shows that influence has became more important after considering the indirect influence. For example, for the candidate "Marine Le Pen", we found that she has 4003 *retweets of Replies* and 37715 *retweets of mentions*, the influence degree obtained after multi-level fusion is still the same degree E.Strong but the belief mass has became more important and has reached 0.99.

Table 9 Results for the top 3 influential French candidates according to multi-level fusion

	Marine Le Pen	**Christine Boutin**	**J.L. Mélenchon**
E.Weak	0	0	0
Weak	0	0	0
Average E	0	0	0.00001235114
Average	0	0	0.00009165825
Strong.E	0	0	0.0003897511
Strong	0	0	0.001821524
V.Strong	0.004328	0.0667283	0, 345777616
E.Strong	0.995672	0.9332717	0.6519071
Ω	0	0	0

Towards ranking users influence

The proposed approach of influence assessment can also be exploited to rank users' influence. In this section, our experimental goal is to detect most influential candidates on the network based on our proposed approach. We focus on the French candidates in the elections, we have 616 candidates with 4 million *tweets*. The following algorithm (Algorithm 2) describes the used method in order to rank users' influence. First, for each candidate we take the influence with maximal pignistic probability (for example, Inf("Marine Le Pen" = E.Strong). After that, we rank candidates by their "maximal influence degree". When two candidates have the same "maximal influence degree":

$$Inf(\text{“Florian Philippot”}) = \text{V.Strong}$$

$$Inf(\text{“Jean-Luc Mélenchon”}) = \text{V.Strong}$$

we compare belief masses of next-greater influence degree:

$$m_{\text{Philippot}}(\text{E.Strong}) > m_{\text{Mélenchon}}(\text{E.Strong})$$

We use the following order of influence degrees ranking:

Algorithm 2: Users influence ranking

Input : Set of users U.
Distribution of pignistic probability for each user.
Output: Users' Ranking R
for $u \in U$ **do**
  $MaxInf$ = Influence degree having maximal pignistic probability;
end
R' = Ranking of U according to $MaxInf$;
if $MaxInf(u) = MaxInf(u')$ **then**
  $MaxInf'$ = Influence degree of next-greater Influence;
end
R'' = Ranking of U according to $MaxInf'$;
$R = R' \cup R$";
return R ;

Ω < V.Weak < Weak < Average.E < Average < Strong.E < Strong < V.Strong < E.Strong

We proceed this way since it is unfair to rank candidates by maximal belief masses they have on the degrees. This is because we may have a user more influential than another although he has a weaker belief mass than him on the same degree. This is due to the fact that, the belief mass on the next-greater degree has increased and became quite important. For example, the candidate "Florian Philippot" has a belief mass on the degree V.Strong weaker than the belief mass of "Jean-Luc Mélenchon" on the same degree as we can see in table 7. Inspite of this, he is ranked before Jean-Luc Mélenchon (table 10) as he has a greater belief mass on the degree E.Strong.

We do the combination procedure for all candidates and deduce their ranking by influence degree. Findings are shown in table 10. The results are general, taking into account the possible patterns in one same measure unlike results shown in table 11. Figure 4 represents candidates' ranking and shows their different belief mass distribution according to the influence degrees.

Table 10 Top influential candidates according to belief fusion

Rank	Candidates	Influence degree	Belief mass
1	Marine Le Pen	E.Strong	0.8173448
2	Florian Philippot	V.Strong	0.5830090
3	Jean-Luc Mélenchon	V.Strong	0.7960571
4	Christine Boutin	V.Strong	0.9796956
5	Aymeric Chauprade	V.Strong	0.4171324655
6	Nicolas Dupont-Aignan	V.Strong	0.5293170700
7	José Bové	V.Strong	0.2925722297
8	Geoffroy Didier	Average	0.2092645352
9	Raquel Garrido	Average	0.2048485
10	Marielle De Sarnez	Average.E	0.2074260

Figure 3 represents a partial visual representation of the diffusion graph corresponding to the French candidates. In order to bypass the visual complexity of the whole graph we use only 1% of all graph data. Big nodes correspond to candidates accounts, small nodes represent other users. The size and colors of main nodes correspond to their influence degrees: red color corresponds to E.Strong, orange corresponds to V.Strong and yellow is for Average and Average.E degree.

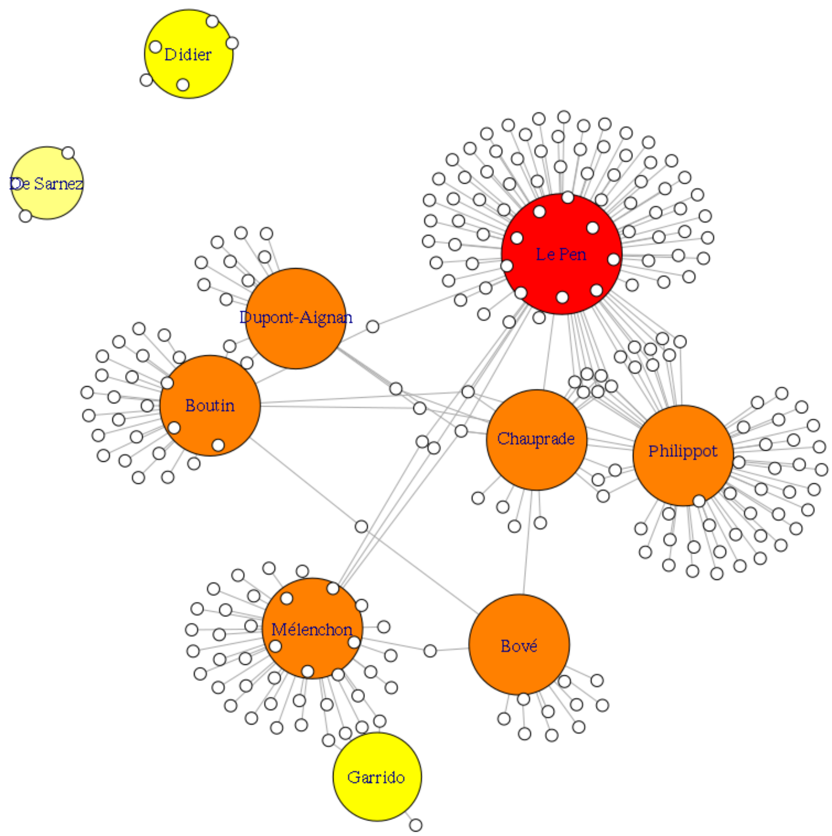

Figure 3 An overview of the influence diffusion graph for french candidates

Table 11 presents the top influential candidates according to the relationships taken individually. They are ranked by their numbers of *retweets*, *mentions* and

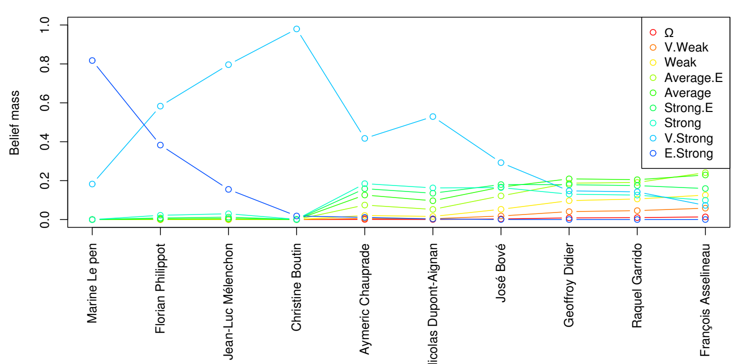

Figure 4 Graphical representation of belief mass distributions for top 10 French candidates

replies. The presented results do not provide the global candidates influence in the network since different rankings for each relationship are used. Users are also ranked according to their centrality degree. It is computed using the number of the candidates' neighbors in the multiplex network. This enables to have a global ranking for the candidates but do not offer any indication on the influence degree of each candidate. We also compared our results with those obtained with the HITS (Table 12), the original algorithm does not combine the influence markers and can be based only on one relationship, we have tested the algorithm with each relationship separately, namely, *retweets*, *mentions* and *replies*. The obtained results show different rankings of the candidates' influence unlike our obtained results.

Table 11 Top influential French candidates according to different relationships and centrality degree

Rank	Retweet	Mention	Reply	Centrality degree
1	M. Le Pen	M. Le Pen	C. Boutin	M. Le Pen
2	F. Philippot	C. Boutin	M. Le Pen	C. Boutin
3	J. Mélenchon	J. Mélenchon	F. Philippot	F. Filippot
4	A. Chauparde	F. Philippot	J. Mélenchon	J. Mélenchon
5	F. Asselineau	N. Dupont-Aignan	L. de Gouyon Matigon	N. Dupont-Aignan
6	C. Morel-Darleux	J. Bove	N. Dupont-Aignan	A. Chauparde
7	N. Dupont-Aignan	A. Chauparde	J. Herpin	J. Bove
8	L. Aliot	R. Garrido	J. Rochedy	G. Didier
9	D. Payre	J. Lavrilleux	G. Didier	R. Garrido
10	Y. Jadot	M. de Sarnez	L. Aliot	Y. Jadot

We also ranked the candidates according to multi-level belief fusion. Table 13 shows the new ranking of candidate's influence after considering indirect influence. Compared to the ranking given in table 10, some candidate's ranking has changed. For exemple, the candidate "Christine Boutin" has became the second most influential candidate as the influence degree increased and became E.Strong with a belief mass of 0.93. This proves the importance of considering the indirect influence in the assessment process.

Conclusion

In this paper, we proposed an influence assessment approach for the *Twitter* social network. This approach addresses limitations of existing systems such as lack of patterns combination and uncertainty ignorance on the given measures. In our work, we proposed a diffusion belief network allowing us to observe different interactions in the network, we considered several patterns: *retweet*, *mention*, *reply*, *retweet of*

Table 12 French candidates ranking according to Hits algorithm

Rank	Reply	Retweet	Mention
1	Marine Le Pen	Marine Le Pen	Marine Le Pen
2	Florian Philippot	Aymeric Chauprade	Aymeric Chauparde
3	Jean-Marie Le Pen	Bernard Monot	Florian Philippot
4	Geoffroy Didier	Florian Philippot	Jean-Marie Le Pen
5	Christine Boutin	Nicolas Bay	Louis Aliot
6	Aymeric Chauparde	Bruno Gollnisch	Bernard Monot
7	Gilles Lebreton	Audrey Guibert	Geoffroy Didier
8	Julien Rochedy	Gilles Lebreton	Julien Rochedy
9	Louis Aliot	Jean-Marie Le Pen	Gilles Lebreton
10	Nicolas Bay	Karim Ouchikh	Bruno Gollnisch

Table 13 Top influential French candidates according to multi-level belief fusion

Rank	Candidates	Influence degree	Belief mass
1	Marine Le Pen	E.Strong	0.995672
2	Christine Boutin	E.Strong	0.9332717
3	Jean-Luc Mélenchon	E.Strong	0.6519071
4	Florian Philippot	E.Strong	0.6021768
5	Aymeric Chauprade	V.Strong	0.66411504
6	Nicolas Dupont-Aignan	V.Strong	0.6615197
7	José Bové	V.Strong	0.6003759
8	Raquel Garrido	V.Strong	0.4805315962
9	Geoffroy Didier	V.Strong	0.416560612
10	Marielle De Sarnez	Strong	0.789653

replies and *retweet of mentions*. Based on belief functions theory, we established a general influence measure for a given user by information fusion of the different patterns. The proposed approach is flexible and can take into account different patterns of interaction in the belief network, the influence measure considers influence exercised on indirected nodes (e.g. a user may *retweet* another's *tweet* indirectly through an intermediate user).

We experimented our approach on real data gathered from *Twitter* in the context of the project TEE 2014. The experiments show that markers combination under uncertainty leads to a quite interesting results.

Interesting perspectives emerge to further strengthen the proposed approach. The method for users ranking will be improved. Besides, we plan to apply the proposed approach on other measures that requires information fusion such as users credibility and *Twitter* styles categorization. And finally, we will consider more complex interaction patterns in the method such as *hashtags* or *favorites* on a multi-relational graph.

Competing interests
The authors declare that they have no competing interests.

Author's contributions
This work is the result of a close joint effort in which all authors contributed almost equally to defining and shaping the problem definition, proofs, algorithms, and manuscript. LA, as the first author, took the lead in composing the first draft of the manuscript, while SK, MS, EL and RF edited it. As such, all authors have read and approved the final manuscript.

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